WELL POSEDNESS AND CONTROL IN MODELS BASED ON CONSERVATION LAWS

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Abstract. Consider a control problem based on a balance law, where a given cost, say an integral functional, needs to be minimized. Once suitable well posedness results are available, the existence of optimal controls easily follows. This presentation overviews several examples of this problem.

Key words. Hyperbolic Conservation Laws, Optimal Control of Conservation Laws

AMS(MOS) subject classifications. 35L65, 49J20

1. Introduction. This presentation is devoted to control problems of the type: maximize $\mathcal{J}(u)$ where u solves

$$
\left\{
\begin{array}{llcl}
\partial_t u + \operatorname{div}_x f(t,x,u) = g(t,x,u) & (t,x) & \in & \mathbb{R}^+ \times \Omega \\
b\left(u(t,x)\right) = \psi(t) & (t,x) & \in & \mathbb{R}^+ \times \partial\Omega \\
u(0,x) = \bar u(x) & x & \in & \Omega\,.
\end{array}
\right.
\tag{1.1}
$$

$\mathcal{J}$ is an integral functional, the flow f and the source term g are sufficiently smooth, $\Omega \subseteq \mathbb{R}^N$ and $u \in \mathbb{R}^n$. The examples below show that control parameters may enter f, g, ψ or $\bar u$. As it is well known, a basic analytical theory for (1.1) is available only when the number n of equations and the space dimension N are in one of the two cases

$$
n \geq 1 \text{ and } N = 1 \qquad \text{or} \qquad n = 1 \text{ and } N \geq 1\,.
$$

2. The Case of Junctions. Note preliminarily that, when $N = 1$, the general setting of (1.1) comprises also the case of junctions. Consider for instance a traffic light, say sited at $x = 0$, separating an incoming road, $x < 0$, from the outgoing one, $x > 0$. Then, the traffic densities u_1, before the traffic light, and u_2, after it, can be assumed to solve the following conservation law at a junction:

$$
\left\{
\begin{array}{llcl}
\partial_t u_i + \partial_x f_i(u_i) = 0 & (t,x) & \in & \mathbb{R}^+ \times \mathbb{R},\ i = 1,2 \\
\Psi\left(u_1(t,0-), u_2(t,0+)\right) = \psi(t) & t & \in & \mathbb{R}^+ \\
u(0,x) = \bar u(x) & x & \in & \mathbb{R}
\end{array}
\right.
\tag{2.1}
$$

where f is the traffic flow, see [36, 53]. The condition Ψ at the junction prescribes the conservation of vehicles as well as other conditions, such as priority rules or the maximization of the junction efficiency, see [53]. The framework of (2.1) comprises also other situations. For instance, u_1 might

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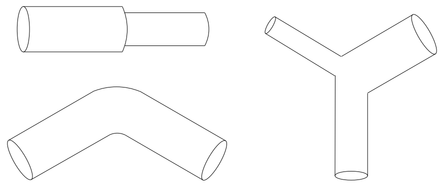
be the vector of the densities of mass and linear momentum of a fluid in a pipe entering a junction or an elbow, sited at $x = 0$, while u_2 is the analogous vector for the fluid in the pipe exiting the junction, with f being the flow of the p-system, see for instance [33, formula (1.1)] or [31, 32, 58]. A key role is played by the choice of the condition Ψ: depending on the specific problem under consideration, it constrains the solution to (2.1) to satisfy specific conditions. In the case of fluid dynamics, Ψ ensures the conservation of mass and that of a component of the linear momentum, see [10, 11, 31, 32, 33], or it may also describe the effect of a compressor at $x = 0$, see [40, 43].

Problem (2.1) is equivalent to (1.1) with $N = 1$, $\Omega = \mathbb{R}^+$, $g = 0$, $u = (u_1, u_2)$, $f = (f_1, f_2)$ and $b(u) = \Psi(u_1, u_2)$, see [40, Proposition 4.2]. More general junctions can be treated similarly, as well as general networks with arcs of finite length. Below, we indifferently refer to initial–boundary value problems like (1.1) or to problems at junctions, like (2.1).

3. Examples based on the *p*-system. Consider a fluid flowing in pipes with constant sections having a common origin at a junction or at an elbow, for instance as one of the three below. In the isentropic, or isothermal, approximation the flow along each pipe is described through the p-system:

$$
\begin{cases}
\partial_t \rho_i + \partial_x q_i = 0 \\
\partial_t q_i + \partial_x P(\rho_i, q_i) = 0
\end{cases}
\qquad
P(\rho, q) = \frac{q^2}{\rho} + p(\rho)
\tag{3.1}
$$



where (ρ_i, q_i) are the density of mass and of linear momentum in the i-th pipe. The pressure is assumed to satisfy standard assumptions, such as [33, **(P)**], typically satisfied by the usual γ-law $p(\rho) = k\,\rho^\gamma$. Remark that q_i is the component of the linear momentum density along the axis of the i-th pipe, so that part of the geometry of the junction can be recovered in (3.1). However, the flow of gas in junctions such as those above is clearly an intrinsically $3D$ phenomenon. Nevertheless, providing a good $1D$ description may significantly shorten numerical integrations. Besides, a full analytical treatment of the p-system in $3D$ is now not available.

Physically, the core of the present $1D$ description lies in the choice of the condition to be imposed on the traces at the junction of the solutions to (3.1), namely

$$\Psi\left((\rho_1, q_1)(t, 0-); (\rho_2, q_2)(t, 0+)\right) = 0\,.$$

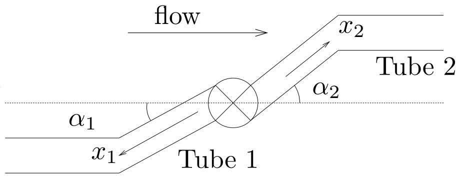
Here, $\Psi\colon (\mathbb{R}^+ \times \mathbb{R})^2 \to \mathbb{R}^2$ is sufficiently smooth. The first component of Ψ ensures the conservation of mass, i.e. $\Psi_1\left((\rho_1, q_1); (\rho_2, q_2)\right) = a_1 q_1 - a_2 q_2$, a_i being the section of the i-th tube. We collect some of the choices found in the current literature for the case of 2 pipes in the table below, from [34], where qualitative properties of the solutions to (3.1) with different conditions at the junction are compared.

Ψ_2	Meaning
$a_1 P(\rho_1, q_1) - a_2 P(\rho_2, q_2)$	Partial conservation of linear momentum, see [32]
$p(\rho_r) - p(\rho_l)$	Equal pressure, typically justified at the static equilibrium, see [10, 11]
$P(\rho_1, q_1) - P(\rho_2, q_2)$	Equal dynamic pressure, see [31, 33]
$a_1 P(\rho_1, q_1) - a_2 P(\rho_2, q_2) + \int_{a_1}^{a_2} p\left(R(\alpha; \rho_l, q_l)\right) \mathrm{d}\alpha$	For two parallel pipes, limit of the condition for smooth variations of the pipes' sections, see [45, 55].

We refer also to [58] for a treatment specific of kinks and to [46, 44] for the full 3×3 system of Euler equations. Above, R is the ρ component of the stationary solution to (3.1), see [34] or [45, Proposition 2.7].

The above structure is relevant from the application point of view, due to its applicability to gas networks and pipelines, see for instance [56, 66, 67, 68, 72]. We consider now in more detail a compressor sited at a junction joining two pipes. Its role is, for example, to pump the fluid up along an inclined pipe as in the figure below.

$$\begin{cases} \partial_t \rho_i + \partial_x q_i = 0 \\ \partial_t q_i + \partial_x P_i = -\frac{\nu q_i |q_i|}{\rho_i} - \rho_i g \sin \alpha_i \end{cases} \qquad P_i = \frac{(q_i)^2}{\rho_i} + p(\rho_i)$$



Here, α_i is the slope of the pipe, ν accounts for friction along the pipe's walls and g is gravity.

As a condition at the junction, a typical choice is that in [72, § 2.2]:

$$q_2(t, 0+) \left(\left(\frac{p\left(\rho_2(t, 0+)\right)}{p\left(\rho_1(t, 0+)\right)} \right)^{(\gamma-1)/\gamma} - 1 \right) = \Pi(t)$$

where the control Π is the power exerted from the compressor. A reasonable lower semicontinuous cost functional on the time interval $[0, T]$ is then

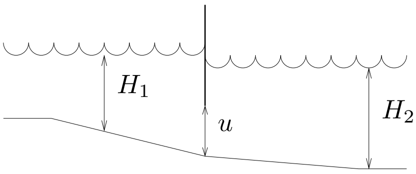
$$\mathcal{J}(\Pi) = \mathrm{TV}\left(\Pi; [0, T]\right) + \|\Pi\|_{\mathbf{L}^\infty([0,T];\mathbb{R})} + \int_0^T \int_a^b \left| p(\rho_2(t, x; \Pi) - \bar{p} \right| \mathrm{d}x \, \mathrm{d}t$$

where $\bar{p}$ is the desired gas pressure along the stretch $[a, b]$ of the second pipe. Here, as usual, $\mathrm{TV}\left(\Pi; [0, T]\right)$ denotes the total variation of the function Π over the time interval $[0, T]$. The first two terms in the right hand side above tend to penalize variations in the compressor power and to minimize its consumption, see [40, 43].

Along a river or a canal, an underflow gate as in the figure below can also be described by the p-system, see [51]:

$$\begin{cases} \partial_t H_i + \partial_x Q_i = 0 \\ \partial_t Q_i + \partial_x P_i = -\frac{\nu Q_i |Q_i|}{H_i} - H_i g \sin \alpha_i \end{cases}$$

$$P_i = (Q_i)^2/H_i + (g/2)(H_i)^2$$



Here, H_i is the height of water in the i-th part of the canal; g, ν and α_i are as above. The system is controlled through u, which is the height of the opening in the gate.

The condition at the junction consists of an equation for the conservation of water together with

$$\big(H_1(t,0-) - H_2(t,0+)\big)\ u(t) = \big(Q_1(t,0-)\big)^2 .$$

Typically, one wants to minimize variations in the water level downstream the gate while ensuring a suitable through flow $\bar{Q}$. Therefore, a reasonable cost functional is

$$\mathcal{J}(u) = \int_0^T \int_a^b \left| Q_2(t,x;u) - \bar{Q} \right| \mathrm{d}x\, \mathrm{d}t + \int_0^T \int_{\mathbb{R}^+} w(x) |\partial_x H_2| .$$

Above, $|\partial_x H_2|$ is the measure theoretic total variation of the space derivative of H_2 and $w \in \mathbf{C}^\infty_\mathbf{c}(\mathbb{R}^+; \mathbb{R}^+)$ is a suitable weight. The cost $\mathcal{J}$ is lower semicontinuous as a function of the solution (H,Q), see [37, Lemma 2.1 and Theorem 2.2].

Several other applications of $1D$ systems of conservation laws are found in the literature. We refer to [21] for a model related to blood flow; to [9, 13, 25, 36] for traffic flow models and to [53] for the treatment of road networks; to [3, 4, 41, 57, 70] for systems describing the movement of granular matter. A wide literature is concerned with the propagation of phase boundaries in fluids, see for instance [1, 26, 64, 71, 74]. In this context, the continuous dependence from the kinetic relation was proved in [29]. This structure also applies to detonation and deflagration phenomena, see for instance [27, 75].

In the case $N = 1$ and $n \geq 1$, the existence of an optimal control in the examples above readily follows, as soon as the solution to (1.1) is proved to depend continuously from the various control parameters. The global in time existence of $\mathbf{BV}$ solutions to $1D$ hyperbolic systems of conservation laws was proved in [54].

The $\mathbf{L^1}$ Lipschitz continuous dependence of the solution from the initial data was proved in the 2×2 case in [18] and in the general case in [14, 19, 20], see also [17, 50].

The $\mathbf{L^1}$ Lipschitz continuous dependence of the solutions from the $\mathbf{L^1}$ distance between boundary data and from the $\mathbf{C^0}$ distance between the boundary profiles was obtained in the 2×2 case in [2]. In the case of a non-characteristic boundary, this result was extended to the $n \times n$ case in [39, 52]. The case of junctions was specifically investigated in [40, 43].

In the case of general hyperbolic conservation laws generating Standard Riemann Semigroups [17, Definition 9.1], the $\mathbf{L^1}$ Lipschitz dependence of the solutions from the $\mathbf{C^0}$ distance between the Jacobians of the flows was obtained in [15].

Most of the above results rely on proving suitable estimates, first on approximate solutions and then passing to the limit. Wave Front Tracking proved to be a very effective procedure to construct approximate solutions to conservation laws, see [17, 50, 59]. Other tools are Glimm scheme, see [54], and vanishing viscosity, see [14]. To pass to balance laws, a standard strategy is based on operator splitting, see [38, § 3.3] or [23, 28, 73]. A general framework that allows to comprise problems with local/non-local sources and/or boundary and/or junctions is in [38, § 3.1] and [39].

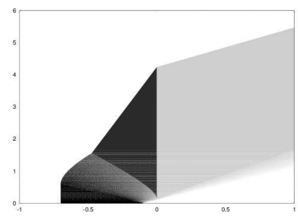
4. 1D Conservation Laws with Unilateral Constraints. The traffic flow along a rectilinear one-way road is often described through the Lighthill-Whitham [65] and Richards [69] model

$$\partial_t \rho + \partial_x f(\rho) = 0 \qquad\qquad f(\rho) = \rho\, v(\rho)\,,$$

where the traffic speed v is assumed to be a known function of the traffic density ρ. A typical choice can be $v(\rho) = V \cdot \big(1 - \rho/R\big)$, V being the maximal speed and R the maximal density, see also [35, **(**R1**)**] for a more general condition on the flow f.

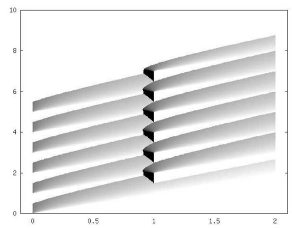
The effect of a toll gate sited at, say, x_r is to limit the flow of traffic below a threshold $q_r = q_r(t)$. We thus obtain the Cauchy problem

$$\left\{\begin{array}{l} \partial_t \rho + \partial_x f(\rho) = 0 \\ \rho(0,x) = \bar\rho(x) \\ f\left(\rho(t,x_r)\right) \leq q_r(t) \end{array}\right. \qquad f(\rho) = V\,\rho\,(1-\rho/R)$$



where the (x,t) diagram above on the right corresponds to $x_r = 0$, $\bar\rho = \chi_{[-0.7,-0.1]}$, $R = 1$ and $V = 1$. From the traffic point of view, it is more realistic to consider the initial boundary value problem with both the inflow and the constraint q_r periodic in time. We thus obtain the following situation:

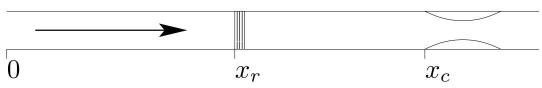
$$\left\{\begin{array}{l} \partial_t \rho + \partial_x f(\rho) = 0 \\ \rho(0,x) = 0 \\ f\left(\rho(t,0)\right) = q_o(t) \\ f\left(\rho(t,x_r)\right) \leq q_r(t) \end{array}\right. \qquad f(\rho) = V\,\rho\,(1-\rho/R)$$



Here, both q_o and q_r vary between 0 and the maximal flow 1/4.

The basic analytical issues concerning the well posedness of these problems are solved in [6, 35]. Various control problems can now be posed, such as minimizing the travel time, see [5] or the variations in the traffic speed, see [37]. Furthermore, assume that road constructions hinder the flow of traffic at x_c, with $x_c > x_r$, see the figure below. Then, describing the effects of the road construction by means of a further unilateral constraint, we are lead to

$$\begin{cases} \partial_t \rho + \partial_x f(\rho) = 0 \\ \rho(0,x) = 0 \\ f\left(\rho(t,0)\right) = q_o(t) \\ f\left(\rho(t,x_r)\right) \leq q_r(t) \\ f\left(\rho(t,x_c)\right) \leq q_c \end{cases}$$



$$f(\rho) = V\,\rho\,(1-\rho/R)$$

An analytical and numerical study of this problem is in [36].

In the study of scalar conservation laws with unilateral constraints, a suitable mixture of Kružkov technique, see [63], with wave front tracking, see [17, 49, 50], proved to be effective, see [6, 35, 36].

5. MultiD Scalar Conservation Laws. A natural application of scalar conservation laws in $2D$ is provided by the modeling of crowd dynamics, see [24, 30, 60, 61]. In this case, ρ is the pedestrian density, which is assumed to solve a conservation law of the type, for instance,

$$\begin{cases} \partial_t \rho + \operatorname{div} f(x,\rho) = 0 & (t,x) \in \mathbb{R}^+ \times \Omega \\ \rho(0,x) = \bar\rho(x) & x \in \Omega \\ f\left(x,\rho(t,x)\right)\cdot \nu(x) = \psi(t) & (t,x) \in \mathbb{R}^+ \times \mathcal{D} \\ f\left(x,\rho(t,x)\right)\cdot \nu(x) = 0 & (t,x) \in \mathbb{R}^+ \times \mathcal{W} \end{cases} \tag{5.1}$$

with $\Omega \subseteq \mathbb{R}^2$, $\nu(x)$ is the interior normal to $\partial\omega$ at x, $\mathcal{D} \subseteq \partial\Omega$ is the door and $\mathcal{W} \subseteq \partial\Omega$ is the wall, with $\mathcal{D} \cup \mathcal{W} = \partial\Omega$ and $\mathcal{D} \cap \mathcal{W} = \emptyset$. $\psi(t)$ is the flow of people entering Ω at time t. Note however that a unilateral constraint $f\left(x,\rho(t,x)\right)\cdot\nu(x) \leq \psi(t)$ might often be more suitable, in particular in the case of people exiting Ω.

Problems of this type apparently received little attention from the mathematical community. In the case of the Cauchy problem

$$\begin{cases} \partial_t \rho + \operatorname{div} f(x,\rho) = 0 \\ \rho(0,x) = \bar\rho(x) \end{cases}$$

the existence of solutions, as well as the fact that the resulting semigroup is non expansive in $\mathbf{L^1}$, was proved in the classical paper by Kružkov [63], whereas the case of bounded domains was dealt with in [12]. However, the stability of solutions with respect to the flow was proved only recently in [47]. Other results in the literature provided similar estimates in the case of particular flows, for instance $f(x,\rho) = a(x)\,b(\rho)$, or required *a priori* bounds on the total variation of the solution, see [16, 22, 62].

Recent results in this direction considers the case $N \geq 1$ and $n = 1$ of (1.1) with a *nonlocal* flow, see [42, 48]. This case is motivated by examples from pedestrian dynamics, see [42, § 4], as well as supply chain management, see [7, 8]. In [42, Theorem 2.10] it is proved that the Cauchy problem for the continuity equation

$$\begin{cases} \partial_t \rho + \operatorname{div} \left(\rho\, V(\rho)\right) = 0 \\ \rho(0,x) = \rho_o(x) \end{cases} \tag{5.2}$$

with $V \colon \mathbf{L^1} \mapsto \mathbf{C^2}$ being a *nonlocal* operator, generates a semigroup S in the sense that $t \mapsto S_t \rho_o$ is the solution to (5.2). Under strong regularity assumptions on v, [42, Theorem 2.10] proves that S_t is differentiable with respect to ρ_o and that its derivative computed at ρ_o in the direction r is characterized by $\left(DS_t(\rho_o)\right) r = \Sigma_t^{\rho_o} r$. Here Σ^{ρ_o} is the semigroup generated by the *linearized* equation

$$\begin{cases} \partial_t r + \operatorname{div} \Big(r\, V(\rho) + \rho \left(DV(\rho)\right)(r)\Big) = 0 \\ r(0,x) = r_o\,. \end{cases}$$

A necessary condition for the optimal control of integral functionals then follows. In [48] another existence result is proved by means also of $\mathbf{L^2}$ techniques.

6. Open Problems. It is clear from the above presentation that several issues, to the present knowledge of the author, are still unanswered.

Concerning the dependence of the solution to (1.1) from the various quantities appearing therein, not all situations have been fully considered, in particular in the case of (5.1).

Concerning optimal control problems, all the results above ensure the existence of such a control. Finding it, either through suitable necessary conditions or through approximate constructive procedures, is a problem still open in many cases when non smooth solutions arise and may develop interacting shocks. The existence of closed–loop, or feedback, controls is also of great interest in most of the examples cited above.

Two other research areas seem worth being considered: the inverse problem and stochastic evolutions. The former is of interest in particular in those situations, such as traffic modeling, in which the various parameters entering the equations are not motivated *a priori* from physics. The latter seems unavoidable when trying to provide a macroscopic description of phenomena that are inherently microscopic, an example being nucleation in phase transitions.

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