Control of Conservation Laws and Reaction-Diffusion Models

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These lectures are devoted to present recent work and some perspectives of research on the following topics:

- Inverse design of conservation laws
- Control of some models of reaction-diffusion type arising in Mathematical Biology and Social Sciences.

The first one is motivated by a number of engineering applications. It can be formulated as the (apparently simple) problem of the backward resolution of the dynamics under consideration, the conservation law. However, since the forward semigroup generates shocks, the backward one is severely ill-posed. Consequently, there are infinitely many initial data that can lead to the same target in finite time. Inspired from recent works by R.M. Colombo and V. Perrolaz [1] and from our collaboration with L. Gosse [2] and T. Liard [3], we will present a fairly complete analytical and numerical solution in one space dimension. We shall also describe some challenging open problems.

The second topic concerns reaction-diffusion models arising in Mathematical Biology and Social Sciences. And we shall present the main results in [4], [5] and [6], together with a summary of the techniques we employ. These models admit a number of relevant particular solutions such as steady states and traveling waves. Furthermore, the state, typically describing a density function, is naturally restricted to take values in the interval [0, 1]. We analyze the problem of controlling the dynamics from one initial special solution to another, preserving these natural bounds, by means of various control functions, entering either as parameters in the nonlinearity or as active boundary controllers. We shall discuss the possible existence of barriers that become an impediment for a number of targets to be achieved, and how this depends on the size of the space domain where the dynamics evolves. We shall conclude this second part describing some of the open problems arising in this fertile field.

References

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