# A Numerical Approach to Interface-Coupled Hyperbolic Evolution

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We consider free boundary value problems with hyperbolic conservation laws in the bulk domains and non-standard transmission conditions across the dynamic interface. By non-standard conditions we mean that it is not possible or feasible to pose the problem in terms of a distributional formulation on the entire spatial domain. Two examples will be adressed in the talk: Inviscid liquid-vapour flow with surface-tension driven interfaces can be described by the compressible Euler equations in the bulk and requires geometric conditions across the interface. Incompressible two-phase flow in porous media with evolving fractures leads to a hyperbolic evolution law in the bulk and notably within the fracture network.

In view of the lacking distributional formulation, numerical discretization methods typically rely on the explicit tracking of the interface. A novel moving mesh approach is introduced that represents the interface via surface elements of the time-dependent bulk domain discretization. The approach is shown to retain conservation and positivity properties on the discrete level. Furthermore it is possible to derive for scalar model problems entropy conditions for the numerical schemes. Numerical simulations in two and three spatial dimensions will be shown.

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