Dissipative Solutions of the Euler Equations

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The Euler equations of gas dynamics are an iconic example of hyperbolic conservation laws. In this talk we introduce generalized solutions of the Euler equations, the so-called dissipative weak solutions. Their existence has been shown by the convergence analysis of suitable, invariant-domain preserving finite volume schemes [1], [2]. In the case that the strong solution of the Euler equation exists, the dissipative weak solutions coincide with the strong solution on its life span. Otherwise, we apply a newly developed concept of K-convergence and prove the strong convergence of the empirical means of numerical solutions to a dissipative weak solution, [3], [4]. The latter is the expected value of the dissipative measure-valued solutions and satisfies a weak formulation of the Euler equations modulo the Reynolds defect measure. In the class of dissipative weak solutions there exists a solution that admits semiflow property and maximizes entropy production. Theoretical results will be illustrated by a series of numerical simulations.

References

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