INdAM Workshop:
Present Research Trends in Conservation Laws

September 8th–10th, 2021

Contents

Scientific Committee

- Giuseppe M. Coclite (Politecnico di Bari)
- Rinaldo M. Colombo (Università degli Studi di Brescia)
- Paola Goatin (INRIA Sophia Antipolis)
- Gabriella Puppo (Università degli Studi di Roma *La Sapienza*)

Organizing Committee

- Maya Briani (IAC-CNR, Roma)
- Francesca Marcellini (Università degli Studi di Brescia)
- Massimiliano D. Rosini (Università degli Studi di Ferrara)

Sponsors

- Istituto Nazionale di Alta Matematica
- Centre Inria Sophia Antipolis - Méditerranée

Program

	Wednesday 8	Thursday 9	Friday 10
08:45-09:00	Opening		
09:00-09:45	Zuazua	Andreianov	Lukáčová
09:45-10:00	Discussion	Discussion	Discussion
10:00-10:20	Piu	Chiarello	Visconti
10:20-10:30	Discussion	Discussion	Discussion
10:30-11:00	Coffee break	Coffee break	Coffee break
11:00-11:45	Garavello	Göttlich	Amadori
11:45-12:00	Discussion	Discussion	Discussion
12:00-12:45	Rohde	Guerra	Klar
12:45-13:00	Discussion	Discussion	Discussion
	Lunch	Lunch	Lunch
14:30-15:15	Tzavaras	Holden	Mascia
15:15-15:30	Discussion	Discussion	Discussion
15:30-15:50	Holle	De Nitti	Sylla
15:50-16:00	Discussion	Discussion	Discussion
16:00-16:30	Coffee break	Coffee break	Coffee break
16:30-17:15	Mishra	Dafermos	Bressan
17:15-17:30	Discussion	Discussion	Discussion
17:30-17:45			Closing

Venue: Hotel Best Western Globus
Viale Ippocrate, 119; 00161 Rome.
Phone: +39 06 4457001.

YouTube link: INdAM Channel.

Zoom link: Please apply to PReTreCoLa@gmail.com.

Updates: Workshop web page.
This program.

Plenary Speakers

BV Solutions to a Hydrodynamic Model of Flocking Type

Debora Amadori
Università degli Studi dell'Aquila

Mathematical models of self-organized systems, that can capture their emergent behavior, have brought new challenges in the mathematical community and a lot of attention in recent years.

In this talk, we will discuss a hydrodynamic model of flocking type with an all-to-all interaction kernel. First, we establish the global existence of entropy weak solutions for arbitrary initial data of bounded variation with finite mass confined in a bounded interval and uniformly positive density therein. Then we show that, under appropriate assumptions, the entropy solution admits time asymptotic flocking.

In collaboration with: Cleopatra Christoforou (University of Cyprus).

To Contents
To Program

Local Compactness for Many-Particle Approximations of LWR Model and Applications

Boris Andreianov
Université de Tours

The talk is devoted to approximation of some discontinuous-flux LWR models by means of follow-the-leader deterministic particle systems. On the basis of local compactness arguments, we prove convergence of approximations in the case of a conservation law with point velocity constraint and in the case of a one-dimensional Hughes model with linear cost. In the first case, this permits to assess the idea that a point velocity constraint should be understood as a flux constraint in the sense of Colombo-Goatin. In the second case, the construction leads to an original existence result for the Hughes model of pedestrian evacuation.

Extensions and alternative arguments are discussed.

In collaboration with: Massimiliano D. Rosini (Università degli Studi di Ferrara/Lublin) and Graziano Stivaletta (Università degli Studi dell'Aquila).

To Contents
To Program

Optimal Control of Propagation Fronts and Moving Sets

Alberto Bressan
Pennsylvania State University

We consider a controlled reaction-diffusion equation, motivated by a pest eradication problem. Our goal is to derive a simpler model, describing the controlled evolution of a contaminated set.

As a first step, we study the optimal control of 1-dimensional traveling wave profiles. Using Stokes' formula, explicit solutions are obtained, which in some cases require measure-valued optimal controls. In turn, this leads to a family of optimization problems for a moving set, related to the original parabolic problem via a sharp interface limit.

The second part of the talk will present results on controllability, existence of optimal strategies, and necessary conditions, together with several examples of explicit solutions.

In collaboration with: Maria Teresa Chiri (Pennsylvania State University) and Najmeh Salehi (Pennsylvania State University).

To Contents
To Program

Damping and the Zero Relaxation Limit in Hyperbolic Systems of Balance Laws

Constantine M. Dafermos
Brown University

The lecture will survey progress and open problems in a research program of constructing BV solutions to hyperbolic systems of balance laws with dissipative source, with an eye to identifying zero relaxation limits.

To Contents
To Program

Well Posedness and Control for Balance Laws Models Inspired by Biology

Mauro Garavello
Università degli Studi di Milano-Bicocca

In the talk we consider a class of hyperbolic balance laws, also known as population balance laws or renewal equations [8]. These are differential equations governing the evolution in time of macroscopic densities, which depend also on *space* variables describing some traits of the individuals, such as age or size structures. Typically, such equations contain nonlocal terms, used for the description of phenomena like natality and interactions between different populations. Various realistic applications can be considered. Some examples are: cell growth [8], cancer dynamics [1], biological resources' models [2], epidemic models for the spreading of a disease, like the COVID-19 [6, 7].

It is thus natural to consider population balance laws both on graphs and on multidimensional spaces. For such models we present some analytic results: existence of a unique solution, continuous dependence with respect to initial conditions, and stability estimates.

Inspired by realistic situations, we propose various control problems, which aim, for example, to minimize the total number of deaths in an epidemic situation [4] or to reduce the mass of cancer cells in a tissue [5]. In specific cases we show the regularity of the input-output map and we prove that optimal controls exist [2, 3].

References

[1] F. Billy, J. Clairambaultt, O. Fercoq, S. Gaubertt, T. Lepoutre, T. Ouillon, and S. Saito. Synchronisation and control of proliferation in cycling cell population models with age structure. *Math. Comput. Simulation*, 96:66–94, (2014).

[2] R.M. Colombo, M. Garavello, Control of Biological Resources on Graphs, *ESAIM: COCV*, vol. 23, pp. 1073–1097, (2017).

[3] R.M. Colombo, M. Garavello, Polynomial Profits in Renewable Resources Management, *Nonlinear Analysis Series B: Real World Applications*, vol. 37, pp. 374-–386, (2017).

[4] R.M. Colombo, M. Garavello, *Optimizing vaccination strategies in an age structured SIR model.* Math. Biosci. Eng. 17, no. 2, pp. 1074-–1089, (2020).

[5] R.M. Colombo, M. Garavello, Well Posedness and Control in Renewal Equations with NonLocal Boundary Conditions, *Mathematical Methods in the Applied Sciences*, to appear.

[6] R.M. Colombo, M. Garavello, F. Marcellini, E. Rossi, *An age and space structured SIR model describing the Covid-19 pandemic.* J. Math. Ind. 10, (2020).

[7] W.O. Kermack, A.G. McKendrick, *A contribution to the mathematical theory of epidemics*, Proc. R. Soc. Lond. A 115, 700-721, (1927).

[8] B. Perthame, Transport equations in biology. Basel: Birkhäuser, (2007).

In collaboration with: R.M. Colombo (Università degli Studi di Brescia), F. Marcellini (Università degli Studi di Brescia) and E. Rossi (Università degli Studi di Modena e Reggio Emilia).

Partially supported by GNAMPA 2019 project: *Equazioni alle derivate parziali di tipo iperbolico o non locale ed applicazioni*.

To Contents
To Program

Non-Local Conservation Laws for Material Flow Problems

Simone Göttlich
Universität Mannheim

The material flow problems under consideration have a granular like structure and allow for a multi-scale model hierarchy. Starting from a detailed microscopic model based on Newton type dynamics, a corresponding macroscopic model is derived, leading to conservation laws with a non-local interaction term. Both modeling approaches are fitted against real data from an experimental setup. In addition to numerical simulation results and theoretical investigations, we address questions of optimal control and extensions to networks.

To Contents
To Program

The Cauchy Problem for a non Strictly Hyperbolic 3×3 System of Conservation Laws Arising in Polymer Flooding

Graziano Guerra
Università degli Studi di Milano-Bicocca

We study the Cauchy problem of a 3×3 system of conservation laws modeling two–phase flow of polymer flooding in rough porous media with possibly discontinuous permeability function. The system loses strict hyperbolicity in some regions of the domain where the eigenvalues of different families coincide, and BV estimates are not available in general. For a suitable 2×2 system, a singular change of variable introduced by Temple [4, 6] could be effective to control the total variation [5]. An extension of this technique can be applied to a 3×3 system only under strict hypotheses on the flux functions [2]. Through an adapted front tracking algorithm we show the existence of solutions for the Cauchy problem under mild assumptions on the flux function, using a compensated compactness argument.

References

[1] A. Bressan, G. Guerra, W. Shen. Vanishing viscosity solutions for conservation laws with regulated flux. *J. Differential Equations*, 266:312–351, (2019).

[2] G.M. Coclite, N.H. Risebro, Conservation laws with time dependent discontinuous coefficients., *SIAM J. Math. Anal.*, vol. 36, pp. 1293–1309, (2005).

[3] G. Guerra, W. Shen, The Cauchy problem for a non strictly hyperbolic 3×3 system of conservation laws arising in polymer flooding., *Commun. Math. Sci.*, to appear, (2021).

[4] E.L. Isacson, J.B. Temple, Analysis of a singular hyperbolic system of conservation laws. *J. Differential Equations* 65, no. 2, pp. 250-–268, (1986).

[5] W. Shen. On the Cauchy problems for polymer flooding with gravitation, *J. Differential Equations*,261(1): 627–653, (2016).

[6] J.B. Temple, Global solution of the Cauchy problem for a class of 2×2 nonstrictly hyperbolic conservation laws. *Adv. in Appl. Math.*, 3(3): 335–375, (1982).

In collaboration with: Wen Shen (Pennsylvania State University).

To Contents
To Program

What are the Compact Sets in Lebesgue Spaces?

Helge Holden
NTNU Norwegian University of Science and Technology

The classical Kolomogorov-Riesz theorem states that a subset of L^p is totally bounded if and only if it is bounded, satisfies a tightness property, and is L^p equicontinuous. We show that boundedness is not needed, but follows from the other assumptions.

Furthermore, we show that this theorem and the classical Arzela-Ascoli theorem both can be derived from a common and simple lemma.

In collaboration with: Harald Hanche-Olsen (NTNU Norwegian University of Science and Technology) and Eugenia Malinnikova (NTNU Norwegian University of Science and Technology)

To Contents
To Program

Kinetic Network Models and their Macroscopic Limits

Axel Klar
FB Mathematik, TU Kaiserslautern
and Fraunhofer ITWM, Kaiserslautern

In this talk we discuss coupling conditions for kinetic models on networks and their macroscopic limits. We consider kinetic models with macroscopic limits ranging from scalar conservation laws, like Burgers' or Lighthill-Witham-type equations, to linear and nonlinear systems of conservation laws, like linearized and nonlinear Euler equations.

Similar to the asymptotic limit of boundary value problems for kinetic models, we consider the limit of the full network problem including the coupling conditions at the nodes. An asymptotic analysis of the kinetic interface layers at the nodes and a matching procedure combining kinetic and macroscopic equations are used to derive macroscopic coupling conditions from the kinetic ones. Numerical simulations illustrate the theoretical results.

In collaboration with: Raul Borsche (TU Kaiserslautern)

To Contents
To Program

Dissipative Solutions of the Euler Equations

Mária Lukáčová–Medviďová
Universität Mainz

The Euler equations of gas dynamics are an iconic example of hyperbolic conservation laws. In this talk we introduce generalized solutions of the Euler equations, the so-called dissipative weak solutions. Their existence has been shown by the convergence analysis of suitable, invariant-domain preserving finite volume schemes [1], [2]. In the case that the strong solution of the Euler equation exists, the dissipative weak solutions coincide with the strong solution on its life span. Otherwise, we apply a newly developed concept of K-convergence and prove the strong convergence of the empirical means of numerical solutions to a dissipative weak solution, [3], [4]. The latter is the expected value of the dissipative measure-valued solutions and satisfies a weak formulation of the Euler equations modulo the Reynolds defect measure. In the class of dissipative weak solutions there exists a solution that is obtained as a vanishing viscosity limit of the Navier-Stokes system [5]. Theoretical results will be illustrated by a series of numerical simulations.

References

[1] E. Feireisl, M. Lukáčová-Medviďová, H. Mizerová: Convergence of finite volume schemes for the Euler equations via dissipative-measure valued solutions, Found. Comput. Math. 20 (2020), no. 4, 923–966.

[2] E. Feireisl, M. Lukáčová-Medviďová, H. Mizerová: A finite volume scheme to the Euler equations inspired by a two velocity approach, Num. Math. 144 (2020), no. 1, 89–132.

[3] E. Feireisl, M. Lukáčová-Medviďová, H. Mizerová: K-convergence as a new tool in numerical analysis, IMA J. Numer. Anal. 40 (2020), no. 4, 2227–2255.

[4] E. Feireisl, M. Lukáčová-Medviďová, B. She, Y. Wang: Computing oscillatory solutions of the Euler system via K-convergence, Math. Math. Models Methods Appl. Sci. 31 (2021), no. 3, 537–576.

[5] E. Feireisl, M. Lukáčová-Medviďová, S. Schneider, B. She: Approximating viscosity solutions of the Euler system, (2021), arXiv:2102.07876

In collaboration with: Eduard Feireisl (Institute of Mathematics of the Czech Academy of Sciences, Prague/Berlin), Hana Mizerová (University in Bratislava), Bangwei She (Institute of Mathematics of the Czech Academy of Sciences, Prague).

Partially supported by TRR 146 Multiscale simulation methods for soft matter systems, TRR 165 Waves to Weather funded by DFG and by the Gutenberg Research College.

To Contents
To Program

Shock Profiles for Fluid-Particles Flows in the Flowing Regime

Corrado Mascia
Università degli Studi di Roma *La Sapienza*

A hydrodynamical limit of a coupled kinetic-fluid model describing the interaction between particles and fluid in a given medium is considered with emphasis on the existence of smooth propagating fronts, i.e. heteroclinic traveling wave solutions. We focus on two different types of models based, respectively, on a Burgers' and an Euler description of the dynamics of the fluid equation exhibiting (some) similarities and (many) differences. The implicit lesson is that the second one is (not surprisingly) more significant even having (surprisingly) the same level of difficulty as the first one.

In collaboration with: Thierry Goudon (INRIA and Universitè Côte d'Azur, Nice, France) and Pauline Lafitte (CentraleSupèlec, Paris, France).

To Contents
To Program

Deep Learning and Computations of Nonlinear Hyperbolic PDEs

Siddharta Mishra
ETH, Zurigo

Deep Learning is being increasingly used in different contexts in Science and Engineering these days. This includes the use of deep learning algorithms in scientific computing. In this talk, I will review some recent progress in applying deep learning based algorithms to accelerate and enhance computations of hyperbolic and related PDEs. In particular, we discuss the use of supervised learning for prediction, uncertainty quantification and constrained optimization of nonlinear systems of conservation laws. The exciting new area of operator learning will also be discussed in this context.

To Contents
To Program

A Numerical Approach to Interface-Coupled Hyperbolic Evolution

Christian Rohde
University of Stuttgart

We consider free boundary value problems with hyperbolic conservation laws in the bulk domains and non-standard transmission conditions across the dynamic interface. By non-standard conditions we mean that it is not possible or feasible to pose the problem in terms of a distributional formulation on the entire spatial domain. Two examples will be adressed in the talk: Inviscid liquid-vapour flow with surface-tension driven interfaces can be described by the compressible Euler equations in the bulk and requires geometric conditions across the interface. Incompressible two-phase flow in porous media with evolving fractures leads to a hyperbolic evolution law in the bulk and notably within the fracture network.

In view of the lacking distributional formulation, numerical discretization methods typically rely on the explicit tracking of the interface. A novel moving mesh approach is introduced that represents the interface via surface elements of the time-dependent bulk domain discretization. The approach is shown to retain conservation and positivity properties on the discrete level. Furthermore it is possible to derive for scalar model problems entropy conditions for the numerical schemes. Numerical simulations in two and three spatial dimensions will be shown.

In collaboration with: Maria Alkämper (University of Stuttgart), Samuel Burbulla (University of Stuttgart) and Jim Magiera (University of Stuttgart).

To Contents
To Program

Existence Theory and Propagation of Oscillations for the System of Viscoelasticity of Strain-Rate Type

Athanatios Tzavaras
KAUST University of Science and Technology of Saudi Arabia

I will review the existence and uniqueness theory for viscoelasticity of Kelvin-Voigt type with non-convex stored energies. The analysis is based on propagation of H^1-regularity for the deformation gradient of weak solutions in two and three dimensions assuming that the stored energy satisfies the Andrews-Ball condition, in particular allowing for non-monotone stresses. it turns out that weak solutions with deformation gradient in H^1 are in fact unique, providing a striking analogy to corresponding results in the theory of 2D Euler equations with bounded vorticity. On the opposite direction, while there is still existence of weak solution for initial data in L^2, there can be propagation of oscillations of the deformation gradient. A counterexample indicates that for non-monotone stress-strain relations in 1-d initial oscillations of the strain lead to solutions with sustained oscillations. Similar phenomena appear in several space dimensions associated with lack of rank-one convexity of the stored energy.

In collaboration with: Konstantinos Koumatos (University of Sussex), Corrado Lattanzio (Università degli Studi dell'Aquila) and Stefano Spirito (Università degli Studi dell'Aquila).

To Contents
To Program

Thresholds for Nonlinear Semigroups and Time-Inversion

Enrique Zuazua
Friedrich-Alexander-Universität Erlangen-Nürnberg, Germany

Some classical nonlinear semigroups arising in mechanics induce unilateral bounds on solutions or threshold phenomena.

Hamilton–Jacobi equations and $1-$d scalar conservation laws are classical examples of such nonlinear effects: solutions spontaneously develop one-sided Lipschitz or semi-concavity conditions.

On the other hand, in practical applications, one is often led to consider the problem of time-inversion, so to identify the initial sources that have led to the observed dynamics at the final time.

In this lecture we shall discuss this problem answering to the following two questions: On one hand, to identify the range of the semigroup and, given a target, to characterize and reconstruct the ensemble of initial data leading to it.

Illustrative numerical simulations will be presented, and geometric interpretations will also be provided.

We shall also present a number of open problems arising in this area and the possible links with reinforcement learning.

To Contents
To Program

Invited Speakers

A Statistical Mechanics Approach to Macroscopic Limits of Car-Following Traffic Dynamics

Felisia Angela Chiarello
Politecnico di Torino

We will derive macroscopic traffic models from car-following vehicle dynamics by means of hydrodynamic limits of an Enskog-type kinetic description. In particular, we will consider the superposition of Follow-the-Leader (FTL) interactions and relaxation towards a traffic-dependent Optimal Velocity (OV) showing that the resulting macroscopic models depend on the relative frequency between these two microscopic processes. If FTL interactions dominate then one gets an inhomogeneous Aw-Rascle-Zhang model, whose (pseudo) pressure and stability of the uniform flow are precisely defined by some features of the microscopic FTL and OV dynamics. Conversely, if the rate of OV relaxation is comparable to that of FTL interactions then one gets a Lighthill-Whitham-Richards model ruled only by the OV function.

References

[1] F.A. Chiarello, B. Piccoli, A. Tosin: A statistical mechanics approach to macroscopic limits of car-following traffic dynamics, (2021), arXiv:2106.15863

In collaboration with: Benedetto Piccoli (Rutgers University), Andrea Tosin (Politecnico di Torino).

To Contents
To Program

Nonlocal-to-Local Singular Limits for Conservation Laws

Nicola De Nitti
Friedrich-Alexander-Universität Erlangen-Nürnberg, Germany

We present some recent results on the problem of approximating a scalar conservation law by a conservation law with nonlocal flux. As convolution kernel in the nonlocal flux, we consider an exponential-type approximation of the Dirac distribution. We prove that the (unique) weak solution of the nonlocal problem converges strongly in $C(L^1_{loc})$ to the entropy solution of the local conservation law.

In collaboration with: Giuseppe M. Coclite (Politecnico di Bari), Jean-Michel Coron (Université Pierre et Marie Curie), Alexander Keimer (University of California, Berkeley) and Lukas Pflug (Friedrich-Alexander-Universität Erlangen-Nürnberg, Germany).

To Contents
To Program

Entropy Dissipation at the Junction for Traffic Flow Models

Yannick Holle
RWTH Aachen University

We are interested in the dynamics of vehicular traffic at a junction using the LWR–model. We aim to derive a junction condition from a kinetic BGK model. Therefore, we impose a kinetic junction condition satisfying a maximum entropy dissipation problem, see also [4]. The kinetic approximation converges towards a solution to the LWR–model satisfying a macroscopic junction condition. The convergence is rigorously justified, similar to [1], using stationary kinetic solutions and Kruzkov entropies.

We illustrate that the obtained macroscopic condition can be defined in different ways. A first definition is based on a Riemann solver satisfying a maximum entropy dissipation problem. In this problem the entropy dissipation at the junction is measured in terms of entropy fluxes. It turns out that this junction condition is equivalent to the condition introduced in [2] if suitable entropy fluxes are chosen. If the entropies in the maximum entropy dissipation problem coincide for all roads, the condition in [1] is obtained.

Finally, we study an example to discuss the role of the entropy fluxes appearing in the maximum entropy dissipation problem. Different entropy fluxes can be used to model different dynamics at the junction.

References

[1] B.P. Andreianov, G.M. Coclite, and C. Donadello. Well-posedness for vanishing viscosity solutions of scalar conservation laws on a network. Discrete Contin. Dyn. Syst., 37(11):5913–5942, 2017.

[2] H. Holden and N.H. Risebro. A mathematical model of traffic flow on a network of unidirectional roads. SIAM J. Math. Anal., 26(4):999–1017, 1995.

[3] Y. Holle. Entropy dissipation at the junction for macroscopic traffic flow models. arXiv: 2105.15003, 2021.

[4] Y. Holle, M. Herty, and M. Westdickenberg. New coupling conditions for isentropic flow on networks. Netw. Heterog. Media, 15(2):605–631, 2020.

In collaboration with: Michael Herty (RWTH Aachen University) and Michael Westdickenberg (RWTH Aachen University).

This work has been funded by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) projects 320021702/GRK2326 Energy, Entropy, and Dissipative Dynamics (EDDy) and HE5386/22-1.

To Contents
To Program

A Microscopic Model for Lane Changing

Matteo Piu
Università degli Studi di Roma *La Sapienza*

In this talk we introduce a microscopic model for traffic flow for multilane roads. The model we propose is based on a single lane model which couples two interaction terms: one from the Optimal Velocity Model by Bando [1], while the second is the classical Follow-the-Leader term [2]. First we describe the model for a single lane, then we describe the extension to the case of two lanes. We study the stability of the steady state solutions for one lane then we illustrate simple lane changing conditions, investigating the steady solutions of the multilane model thus obtained. Finally, we study the linear stability of the multilane model, comparing the results with numerical simulations carried out with the microscopic model. The numerical results confirm the predictions of the linear stability analysis. The numerical tests also show that the model is able to reproduce Stop & Go waves, a typical feature of congested traffic.

References

[1] M. Bando, K. Hasebe, A. Nakayama, A. Shibata and Y. Sugiyama, (1995). Dynamical model of traffic congestion and numerical simulation. Physical Review E, 51(2), 1035-1042.

[2] L.A. Pipes, (1953). An operational analysis of traffic dynamics. Journal of Applied Physics, 24(3), 274-281.

In collaboration with: Gabriella Puppo (Università degli Studi di Roma *La Sapienza*).

To Contents
To Program

Inverse Design for Compactly Heterogeneous Conservations Laws

Abraham Sylla
Institut Denis Poisson, Université de Tours

We present an extension of the results obtained by Colombo and Perrollaz [1] regarding the set of inverse designs for a class of scalar conservation laws with compact space dependency. The key ingredients are the notion of generalized characteristics of Dafermos [2] and the correspondence with the associated Hamilton-Jacobi equation.

References

[1] R.M. Colombo and V. Perrollaz: Initial data identification in conservation laws and Hamilton-Jacobi equations, Journal de Mathématiques Pures et Appliquées, **138**:1–27, 2020.

[2] C.M. Dafermos. Generalized characteristics and the structure of solutions of hyperbolicconservation laws. Indiana University Mathematics Journal, **26** (6):1097–1119, 1977.

In collaboration with: Rinaldo M. Colombo (Università degli Studi di Brescia) and Vincent Perrollaz (Institut Denis Poisson, Université de Tours).

To Contents
To Program

On Continuum Limits of the Ensemble Kalman Inversion

Giuseppe Visconti
Università degli Studi di Roma *La Sapienza*

The Ensemble Kalman Filter (EnKF) belongs to the class of iterative particle filtering methods and can be used for solving control–to–observable inverse problems. In this context, the EnKF is known as Ensemble Kalman Inversion (EKI) [1]. In recent years several continuous limits in the number of iteration [2] and particles [3] have been performed in order to study properties and reduce the complexity of the method. These limits lead to a high–dimensional conservation law which provides a statistical description of the solution of an inverse problem.

Linear stability analysis reveals a possible instability of the solution provided by the continuous limits of the EKI. In this talk, inspired by [5], we discuss how to address this issue by introducing a stabilization of the dynamics at the continuous level which leads to a method with globally asymptotically stable solutions. We illustrate theoretical results and the performance of the stabilized version by using test inverse problems from the literature.

References

[1] M. Iglesias, K. Law and A.M. Stuart. Ensemble Kalman Methods for Inverse Problems. *Inverse Probl.*, 29(4):045001, 2013.

[2] C. Schillings and A.M. Stuart. Analysis of the Ensamble Kalman Filter for Inverse Problems. *SIAM J. Numer. Anal.*, 55(3):1264–1290, 2017.

[3] M. Herty and G. Visconti. Kinetic Methods for Inverse Problems. *Kinet. Relat. Models*, 12(5):1109–1130, 2019.

[4] M. Herty and G. Visconti. Continuous limits for constrained ensemble Kalman filter. *Inverse Probl.*, 36(7):075006, 2020.

[5] D. Armbruster, M. Herty and G. Visconti. A Stabilization of a Continuous Limit of the Ensemble Kalman Inversion. *Submitted*, arXiv:2006.15390, 2021.

In collaboration with: Dieter Armbruster (Arizona State University), Michael Herty (RWTH Aachen University).

To Contents
To Program